Multilayer Thermionic Refrigerator and Generator

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Abstract

A new method of refrigeration is proposed. Cooling is obtained by thermionic emission of electrons over periodic barriers in a multilayer geometry. These could be either Schottky barriers between metals and semiconductors or else barriers in a semiconductor superlattice. The same device is an efficient power generator. A complete theory is provided.

A new method is proposed for refrigeration and power generation. The devices are composed of multilayers of periodic barriers. The currents are perpendicular to the barriers. Such devices are presently under consideration for thermoelectric cooling and power generation. Here we propose that the same devices can be used for efficient thermionic cooling, or power generation.

What is the difference between a thermoelectric device[1, 2] and a thermionic one[3, 10]? Their descriptions are remarkably similar. In both cases one puts a temperature gradient on a semiconductor. Both devices are based upon the idea that electron motion is electricity. But the electron motion also carries energy. Forcing a current transports energy for both thermionic and thermoelectric device. The basic difference seems to be whether the current flow is ballistic or diffusive. In thermionic motion, the device has relatively high efficiency if the electrons ballistically go over and across the barrier. They carry all of their kinetic energy from one electrode to the other. In thermoelectric devices, the motion of electrons is quasiequilibrium and diffusive. One can describe the energy transport by a Seebeck coefficient, which is an equilibrium parameter. There have also been recent theories of nonequilibrium thermoelectric effects[11]. They seem to be midway between the thermionic(ballistic) and thermoelectic(quasiequilibrium) regimes. We note that many semiconductor superlattices, with short periods, are being made[12] and measured along the c-axis. These periods are so short that electron motion over the barrier is probably ballistic. We suggest that these devices cool by thermionic effects rather than thermoelectric ones.

Earlier we suggested[3] that thermionic devices could be used as refrigerators. For the usual device of two metal plates separated by an air gap, cooling at room temperature requires a low work function. No metals have values that low ($\phi \sim 0.3$ eV). Numerous groups have suggested[4]-[10] that such small barriers are easily attainable in semiconductor systems. Here the barriers are semiconductors, while the electrodes could be either metals or other semiconductors. We considered this geometry in our original paper, but thought that the thermal conductivity of the solid would be a major obstacle. Here we show that the thermal effects can be dealt with by going to a multilayer geometry. We show that such devices have efficiencies which could be twice those of thermoelectric ones.

The physics behind thermionic cooling is simple. Most physicists are familiar with the technique of cooling liquid helium-4 by pumping the vapor from the cryostat. The most energetic helium atoms leave the liquid and

become gas molecules. Pumping them away removes these energetic atoms, thereby cooling the liquid. In thermionic refrigeration, one uses a voltage to sweep away the most energetic electrons from the surface of a conductor. Those electrons with sufficient energy to overcome the work function are taken away to the hot side of the junction. Removing the energetic electrons from the cold side cools it. Charge neutrality is maintained at the cold side by adding electrons adiabatically through an ohmic contact.

Thermionic devices must have the electrons ballistically traverse the barrier in order to have a high efficiency. This requires that the mean-free-path λ of the electron in the barrier be longer than the width L of the barrier. This constrains the barrier width L to be rather small. This fact is a key feature of the analysis. The general constraints are

$$\lambda > L > L_t \tag{1}$$

$$L_t = \frac{\hbar}{2k_BT}\sqrt{\frac{e\phi}{m^*}} \tag{2}$$

where L_t is the minimum thickness to prevent the electron from tunneling through the barrier. Values of L_t seem to be around 5-10 nm for most semiconductors. The values of λ are less certain. There are good measurements of the electron mean-free-path for motion along the layers, but little data for motion perpendicular to the layers. If these values are similar, then one can easily find that $\lambda \sim$ 50-100 nm. Thus there is room in the above inequality to find values of L which work.

The general form of the energy currents for a single barrier are

$$J_Q = J_{Qe} - \frac{\delta T}{R_1} \tag{3}$$

$$R_1 = 2R_I + \frac{L}{K} + \frac{L_e}{K_e} \tag{4}$$

The first term in the energy current is the electron part from thermionic emission. It is given below. The second term is the phonon part, which contains the thermal resistance R_1 for one barrier. The thermal resistance depends upon the thickness (L_e) and thermal conductivity (K_e) of the electrodes, as well as two interface ('Kapitza') terms R_I[13, 14]. The largest term will usually be L/K for the semiconductor barrier. The problem is that L is small which makes R_1 small which makes $\delta T/R_1$ big. This is the problem with the

thermal conductivity. It can be overcome by having δT be small. This is the reason for the multilayer geometry. Each barrier can support only a small temperature difference δT_i. A macroscopic temperature ΔT is obtained by having N layers so that $\Delta T = N\langle\delta T_i\rangle$. Most of our modeling assumes that δT_i has an average value of 1-2 °C. Below we show that adequate cooling power is available for these values.

1 Refrigeration

1.1 Single Barrier

The first step in the derivation is to solve for the currents over a single barrier of width L. We assume the barrier is a constant at zero applied voltage, which means it has a square shape. At a nonzero voltage δV the barrier has the shape of a trapezoid. The formulas for the electrical (J) and heat (J_Q) currents are given in terms of the hot and cold temperatures (T_h, T_c)[3]

$$
\begin{aligned}
J_{Rj} &= AT_j^2 e^{-e\phi/k_BT_j} && (5)\\
A &= \frac{emk_B^2}{2\pi^2\hbar^3}\mathcal{T} && (6)\\
J &= J_{Rc} - J_{Rh}e^{-e\delta V/k_BT_h} && (7)\\
eJ_Q &= [e\phi + 2k_BT_c]J_{Rc} && \\
&- [e\phi + 2k_BT_h]J_{Rh}e^{-e\delta V/k_BT_h} - \frac{\delta T}{R_1} && (8)\\
\delta T &= T_h - T_c && (9)
\end{aligned}
$$

These equations are for a single barrier. Eqn.(5) is the standard Richardson's equation[15] for the thermionic current over a work function $e\phi$ which in this case is the Schottky barrier height between the metal and semiconductor. Alternately, it is the barrier in a semiconductor quantum well. The factor of $\mathcal{T}$ denotes the fraction of electrons transmitted from the metal to the semiconductor. It is calculated using quantum mechanical matching of the wave functions. The formulas for J and J_Q assume the bias $e\delta V$ is to lower the Fermi level on the hot side, so that the net flow of electrons is from cold to hot.

For electrons the charge e is negative which makes ϕ, J also negative. We find this confusing to treat, so we take e and ϕ as positive as if the system

were a hole conductor. Then $J > 0$ for particle flow to the right, which we are assuming.

We show that for a single layer the optimal value of applied bias $e\delta V \propto \delta T$ which is also small. Denote as T the mean temperature of the layer, and then $T_c = T - \delta T/2, T_h = T + \delta T/2$. Then we expand the above formulas for the currents in the small quantities $(\delta T/T, e\delta V/k_B T)$ and find, after some algebra

$$
\begin{aligned}
J &= \frac{eJ_R}{k_B T}[\delta V - V_J] && (10)\\
J_Q &= J_R(b+2)[\delta V - V_Q] && (11)\\
b &= \frac{e\phi}{k_B T} && (12)\\
eV_J &= k_B \delta T[b+2] && (13)\\
eV_Q &= k_B \delta T q && (14)\\
q &= b + 2 + u && (15)\\
u &= \frac{2+Z}{b+2} && (16)\\
Z &= \frac{e}{k_B R_1 J_R} = Z_0 e^b && (17)\\
Z_0 &= \frac{e k_B}{R_1 A (k_B T)^2 \mathcal{T}} = \left(\frac{T_R}{T}\right)^2 && (18)\\
(k_B T_R)^2 &= \frac{2\pi^2\hbar^3}{m k_B R_1 \mathcal{T}} && (19)
\end{aligned}
$$

The symbol J_R denotes Richardson's current eqn.(5) at temperature T. The dimensionless constant Z_0 plays an important role in the results. Refrigeration requires that $\delta V > V_Q$. High efficiency requires that $Z_0 < 1$. We have also introduced the dimensional constant T_R which is determined by the thermal resistance and the effective mass.

The efficiency η of a single barrier is defined as the heat flow divided by the power input

$$
\begin{aligned}
\eta &= \frac{J_Q}{\delta V J} && (20)\\
&= \frac{k_B T(b+2)}{e} \frac{\delta V - V_Q}{\delta V(\delta V - V_J)} && (21)
\end{aligned}
$$

We now vary δV in the above equation to find the value of $\delta V = V_m$ which gives the maximum efficiency η_m which is

$$V_m = V_Q + \sqrt{V_Q(V_Q - V_J)} \tag{22}$$

$$\eta_m = \left(\frac{T}{\delta T}\right)\hat{\eta} \tag{23}$$

$$\hat{\eta}(\phi) = \frac{b+2}{(\sqrt{q}+\sqrt{u})^2} \tag{24}$$

The first factor on the right in eqn.(23) is approximately the Carnot efficiency of the one-layer system. The remaining factor $\hat{\eta}$ is the reduction of this efficiency due to the inefficiency of the device. The latter factor depends upon the parameter $b = e\phi/k_BT$ and the dimensionless factor Z_0 contains the thermal resistivity R_1. Fig. 1 shows the function $\hat{\eta}(\phi)$ for several values of T_R in the range between 100 and 500 K. It is important to have $\hat{\eta}$ be as large as possible. Clearly this constraint limits the value of T_R to smaller values, which has Z_0 to be less than one. For larger values of Z_0 the effective efficiency becomes too small to be useful.

The value of T_R is reduced by the Kapitza resistance R_I at the interfaces. Numerical estimates for R_I are given by[13, 14]. The device will only be efficient when R_I large. Large thermal boundary resistance can be achieved by having acoustic mismatch between the metal and semiconductor layers. The important point is that the thermal conductivity term has to stay small. The thermal resistivity is large in semiconductor superlattices[16, 17, 18, 19, 20, 21].

The results for the single barrier demonstrate that one has to have small values of barrier height $e\phi \approx 2k_BT$ and large values of the thermal resistance. These characteristics carry over to the theory of the multilayer device. Small values of the barrier heights are reported in several references[12, 22, 23, 24]

1.2 Multiple Barriers

Now consider the multilayer thermionic refrigerator. There are N barriers, with alternating electrodes. Thus there are $2N+1$ layers, assuming electrodes come first and last. Denote δT_i and δV_i as the temperature and voltage change across one barrier. These values change from the initial to the final layer. This change is due to the generation and flow of heat. At each barrier,

the electron ballistically crosses the barrier region, and then loses an amount of energy $e\delta V_i$ in the electrode. The heat generated at each electrode must flow out of the sample according to the equation

$$\frac{d}{dx}(J_{Qi}) = J\frac{\delta V_i}{L_i} \tag{25}$$

where L_i is the effective width one one barrier plus one metal electrode. We take L_i to be a constant, although it could vary with i. The width of the device is $D = NL_i$. In normal usage there would be a negative sign on the right-hand side of this equation. It is absent since we changed the sign of J.

The functions change slowly with i and we treat them as continuous variables $\delta T_i/L_i = dT/dx$ and $\delta V_i/L_i = dV/dx$. The variation of δV_i upon i is unknown, but we guess that it is proportional to δT_i. Introduce the dimensionless function $v(x)$

$$e\delta V_i = k_B \delta T_i v \tag{26}$$

$$J = J_R\frac{\delta T_i}{T}[v - b - 2] \tag{27}$$

$$eJ_Q = J_R(b+2)k_B\delta T_i[v - q] \tag{28}$$

We use eqn.(27) to define δT_i since the current J is the only constant among all of the parameters

$$\delta T = \frac{JT}{uJ_R}I \tag{29}$$

$$I = \frac{u}{v-b-2} \tag{30}$$

$$D\frac{dT}{dx} = \frac{ITyZ}{u} \tag{31}$$

$$y = J\left(\frac{k_B}{e}\right)R_N \tag{32}$$

$$R_N = NR_1. \tag{33}$$

The variable $I(x)$ is introduced in eqn.(29,30). After trying various alternatives, we decided it is the convenient variable for the present problem. The parameter y is a dimensionless current. Eventually we vary this parameter to find the optimal efficiency. The thermal resistance of the entire device is

denoted as R_N. Its value is irrelevant since it ocurs in y which is a variational parameter. Use eqn.(29) to evaluate δV_i in (26) and this result is inserted into eqn.(25)

$$
\begin{aligned}
J_Q &= J(\phi + 2\frac{k_B T}{e})(1-I) && (34)\\
D\frac{d}{dx}(I) &= -y\frac{Z}{2+Z}[u + I(b+2I)] && (35)\\
D\frac{dT}{dx} &= \frac{yITZ}{u} && (36)
\end{aligned}
$$

Eqns.(35) and (36) describe the variation in temperature and voltage across the multilayer device. The input parameters are the barrier height $e\phi(b = e\phi/k_B T)$ and the parameter T_R which depends on the thermal resistance. The parameter y is variational. One solves the equations for different values of y and chooses the value which give the best results. One selects an initial guess for $I(x=0) = I_c$ at the cold end along with $T(0) = T_c$. Then one can iterate the above two equations using one sided derivatives. The value of I_c is varied until one finds the desired value of T_h. The calculation is done for different values of y until the maximum efficiency is attained. The Carnot efficiency is defined as the heat taken from the cold side J_{Qc} divided by the input power $J\Delta V$

$$
\begin{aligned}
\eta &= \frac{J_{Qc}}{J\Delta V} && (37)\\
J_{Qc} &= J\left(\frac{k_B T_c}{e}\right)(b_c + 2)[1 - I_c] && (38)\\
\Delta V &= \sum_{i=1}^{N} \delta V_i = \left(\frac{yk_B}{eN}\right)\sum_i \frac{T_i Z(T_i)}{u_i}[u + I_i(b_i + 2)] && (39)
\end{aligned}
$$

where ΔV is the net voltage drop across the multilayer device. The above integral for ΔV is easy after one has solved for $T(x)$ and $I(x)$.

Fig.2 shows the efficiency of a refrigerator, with T_c=260 K and T_h= 300 K, for four values of $T_R = 100 - 500K$. They are shown as a function of of ϕ. The efficiencies are quite high: η=2.5 for $T_R = 100$ K to η=0.6 for $T_R = 500$ K. The comparable value for the thermoelectric refrigerator, with a dimensionless figure of merit equal to unity, is $\eta_{TE} = 0.70$. That is

comparable to the thermionic results when $T_R = 500$ K. Thus one wants to operate the thermionic device with values of T_R less than 500 K.

The factor of T_R is determined by the thermal resistivity. Reducing the thermal resistivity by a factor of 25 only reduces the efficiency by a factor of 5. Thus the efficiency scales with T_R rather than with R_1. This is quite different than in thermoelectric devices, where the result is almost directly proportional to the thermal resistivity. Note in eqn.(35) that Z enters in the formula in the combination of $Z/(2+Z)$ which saturates at large values of Z. The increase is Z_0 is offset by a lower value of barrier height ϕ.

The exact results show that the thermionic refrigerator has about 30% of the ideal Carnot efficiency $\Delta T/T_c$ for values of T_R = 200-300 K. Our calculations show that the efficiency slowly declines with increasing values of T_R. The value of ϕ where the maximum efficiency occurs also declines with increasing value of T_R.

2 Multilayer Thermionic Power Generation

Power generation can be understood by considering a simple device with one barrier between two electrodes. If the electrodes are at different temperatures, and if there is no initial voltage between the electrodes, then electrons are thermally excited over the barrier. A net electron flow goes from hot to cold. If the electrodes are insulated, then the electrodes become charged and the system develops an open circuit voltage, which opposes the flow of electrons and reduces it to zero. Connecting the electrodes to an external circuit causes current to flow. Power can be extracted from the device. The behavior is identical in concept to a solar cell. Here the efficiency is calculated using the usual definition: it is the external power $J\Delta V$ divided by the heat extracted from the hot electrode J_{Qh}.

2.1 One Barrier

Again the barriers are assumed to be thinner than the mean-free-path of the electrons, so that one can apply the formulas of thermionic emission. There is a small temperature drop δT and voltage drop δV across the barrier. The

efficiency of a single barrier using eqns.(10)–(14) is

$$\eta = \frac{e}{k_B T(b+2)} \frac{\delta V(\delta V - V_J)}{\delta V - V_Q} \tag{40}$$

The voltage δV is varied to find the value δV_m which gives the maximum efficiency η_m

$$\delta V_m = \frac{K_B \delta T}{e} \sqrt{q}(\sqrt{q} - \sqrt{u}) \tag{41}$$

$$\eta_m = \frac{\delta T}{T} \hat{\eta} \tag{42}$$

where $\hat{\eta}$ is defined in eqn.(24). It is the same function which is found in the efficiency of the refrigerator.

2.2 N-Barriers

Again assume that the device has $2N + 1$ layers. Barriers alternate with electrodes, with electrodes on both ends. The electrodes can be metals, or else conducting semiconductors. The barriers are going to be nonconducting semiconductors. Again we shall find that the barrier height $e\phi$ is going to be small. There is a small temperature drop δT_i and voltage drop δV_i across each barrier i. Eqns.(10)-(14) also apply to the generator. The difference from the refrigerator is that now $J < 0$ and $J_Q < 0$. This makes the parameter y be negative. Another change is that $b + 2 > v$ so that the function $I(x)$ is negative. Thus we can apply eqns.(35,36) with the understanding that y and $I(x)$ are both negative.

The formula for the efficiency is the inverse of the one for the refrigerator

$$\eta = \frac{e\Delta V}{(e\phi + 2k_B T_h)(1 - I_h)} \tag{43}$$

These equations were solved on the computer. We assumed that $T_c = 300$ K and $T_h = 400$ K, so that $\Delta T = 100$ K. Figure 3 shows the efficiency as a function of barrier height for several values of T_R. This should be compared with a thermoelectric generator which has an efficiency of η=0.048 for the same operating temperatures. Clearly the thermionic device is more efficient for small values of T_R. Figure 4 shows the temperature and δV_i profiles along the thermionic device, assuming that $N = 100$.

3 Thermoelectric Analogy

Thermionic devices are not thermoelectric devices. However, it is useful to examine the analogy between the two kinds of devices. Once we linearize the eqns. (10,11,25) for small voltage drops, and then make them continuous in a multilayer geometry, they become identical in form to the equations of a thermoelectric. This analogy gives the effective formulas for the conductivity σ, the Seebeck coefficient S, and the thermal conductivity K.

$$\sigma = \frac{eJ_R L_i}{k_B T} \tag{44}$$

$$S = \frac{k_B}{e}(b+2) \tag{45}$$

$$K = [\frac{k_B}{e} J_R + \frac{1}{R_1}] L_i \tag{46}$$

$$\mathcal{Z} = \frac{\sigma S^2 T}{K} = \frac{b+2}{u} \tag{47}$$

The first term in the thermal conductivity is the electronic contribution K_e. Note that it obeys a modified Wiedemann-Franz Law

$$K_e = \sigma T \left(\frac{k_B}{e} \right)^2 \tag{48}$$

The last line in eqn. (47) gives the dimensionless figure of merit $\mathcal{Z}$. It is known in thermoelectric devices that the maximum efficiency is obtained by having $\mathcal{Z}$ be as large as possible. Varying b to maximize $\mathcal{Z}$ gives the relationship

$$be^b = \frac{4}{Z_0} = \frac{2mk_B R_1}{\pi^2 \hbar^3 (k_B \bar{T})^2} \tag{49}$$

where $\bar{T}$ is the mean temperature in the device. This is an accurate estimate of the optimal barrier height ϕ, where $b = e\phi/k_B\bar{T}$. Furthermore, the thermoelectric estimates of the efficiency of the refrigerator (η_r) and generator (η_g)

$$\eta_r = \frac{\gamma T_c - T_h}{\Delta T(\gamma + 1)} \tag{50}$$

$$\eta_g = \frac{\Delta T(\gamma - 1)}{\gamma T_h + T_c} \tag{51}$$

$$\gamma = \sqrt{1 + \mathcal{Z}} \tag{52}$$

are remarkably accurate compared to the computer solutions. The Eqs. (50) and (51) estimate the correct efficiency with an accuracy of about 1%. These formulas provide analytical estimates of the efficiency. One can also use the thermoelectric analogy to find the current densities at maximum efficiency as well as other parameters.

In thermoelectric devices it is highly desirable but rather rare to have values of $\mathcal{Z} > 1$. However, in modeling thermionic devices we find values of $\mathcal{Z}$ much larger than one for reasonable values of thermal resistance. The effective Seebeck coefficient in eqn.(45) is about 250-300 μV/K since values of b are between one and two. By using the low values of thermal conductivity reported along the c-axis of a superlattice, we estimate that T_R = 200-400 K. For this range of parameters the effective dimensionless figure of merit $\mathcal{Z}$ is between 2 and 5. The efficiencies of the thermionic devices are correspondingly much higher than in thermoelectric devices. Ballistic transport carries more heat than diffusive flow.

4 Discussion

A detailed theory is presented of the properties of a multilayer thermionic refrigerator and power generator. It is showns that if the thermal resistance is high, that the devices can be twice the efficiency of the equivalent thermoelectric devices. The values of thermal resistiviity needed to make them work well are in the range of reported values for multiple quantum wells. It appears these devices are practical.

One of the interesting questions is to select the value of N. This determines the number of multilayers. As long as this number is larger than about ten in refrigerators, it does not change the efficiency. The choice of N may affect the thermal resistance. For a fixed temperature drop ΔT, the value of N affects the average temperature drop per layer $\langle \delta T \rangle = \Delta T/N$. The cooling power is the energy current from the cold side. An estimate is

$$J_{Qc} = \frac{A\bar{T}\phi\Delta T}{N} \exp(-e\phi/k_B\bar{T}) \tag{53}$$

where $\bar{T}$ is the average temperature of the device. For a refrigerator take $A = 120$ A/(cm^2 K^2) along with $\bar{T} = 280$ K, $\phi = 0.050$ eV, and $\langle \delta T \rangle = 1$K. Then the result is $J_{Qc} = 212$ W/cm^2. This is a large cooling power. Thus having a small temperature drop permits ample cooling. It does require a small barrier height. Incidently, the numerical solutions allow an accurate calculation of J_{Qc} and the above formula is an underestimate by about a factor of two or three. One could increase N by a factor of ten and still have ample cooling power. The manufacturing costs would depend on N. Another issue is mechanical strength. A thicker device (larger N) is stronger. Thus there are trade offs between cooling power, manufacturing costs, and mechanical strength. Such engineering issues are beyond the scope of the present discussion.

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Appendix

Here we examine the transport of electricity and heat using the drift-diffusion eqn.(1). We will show that the amount of heat is negligibly small when one uses this equation. Unless the layer thickness is in the regime where thermionic emission is valid, the semiconductor barrier does not transport significant amounts of heat.

We consider a square barrier for the semiconductor. Assume there is a small applied potential and a small temperature difference $\delta T = T_h - T_c$. Then the formula for the current should be

$$J = e\mu n \frac{\delta V}{L} - \mu k_B T \left[\frac{n(L) - n(0)}{L} \right] \tag{54}$$

$$n(0) = n_0 e^{-e\phi/k_B T_c} \tag{55}$$

$$n(L) = n_0 e^{-e\phi/k_B T_h} \tag{56}$$

$$n = n_0 e^{-e\phi/k_B T} \tag{57}$$

We expand the density exponents using $T_{h,c} = T \pm \delta T/2$ and find

$$J = \frac{e\mu n}{L}[\delta V - V_D] \tag{58}$$

$$V_D = bk_B\delta T \quad (59)$$

The above formula resembles eqn.(10). The two constant voltages V_J and V_D are similar. The prefactors are very different. Denote by r the ratio of the two prefactors, which gives the ratio of the magnitudes of the currents predicted by drift-diffusion compared to the currents predicted by thermionic emission

$$r = \frac{e\mu n}{eJ_R/(k_BT)} \quad (60)$$

$$= \frac{\sigma}{\sigma_0}\left(\frac{L_0}{L}\right)e^b \quad (61)$$

$$\sigma_0 = \frac{e^2 m L_0 k_B T}{2\pi^2\hbar^3} \quad (62)$$

where σ is the conductivity of the semiconductor and L_0 is a characteristic length which we take to be one micron. At room temperature we find that $\sigma_0 = 8.1$ kS/m. InSb has a mobility of 8 $\text{m}^2/(\text{V s})$ at $n = 10^{20}/\text{m}^3$ for $\sigma = 130$ S/m. Even for $b = 4$ this predicts a ratio of $r \approx 10^{-3}$. So the current due to drift-diffusion is one thousand times smaller than the current due to thermionic emission. The energy currents have the same ratio of proportionality. Thus the flow of heat is negligible when the thickness of the semiconductor is greater than the mean free path of the electron, so that the particles diffuse.

References

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[1] G. Mahan, B. Sales and J. Sharp, *Physics Today*, (March 1997) pg 42

[2] G. D. Mahan, *Solid State Physics* **51**, ed. H. Ehrenreich and F. Spaepen (Academic Press, 1998) pg 81

[3] G. D. Mahan, J. Appl. Phys. **76**, 4362 (1994)

[4] D. M. Rowe and G. Min, Proc. 13th Int. Conf. Thermoelectrics, 1994, pg 339

[5] V. N. Bogomolov, D. A. Kurduykov, A. V. Prokofiev, Yu. I. Ravich, L.A. Samoilovich and S. M. Samoilovich, Proc. 14th Int. Conf. Thermoelectrics 1995

[6] B. Moyzhes, Proc. 15th Int. Conf. Thermoelectrics, 1996, pg 183

[7] G. Kainz (unpublished)

[8] Y. Nishio (unpublished)

[9] A. Shakouri and J.E. Bowers, Appl. Phys. Lett. **71**, 1234 (1997)

[10] A. Shakouri, E. Y. Lee, D. L. Smith, V. Narayanamurti and J. E. Bowers, *Microscale Thermophysical Engineering* (in press)

[11] V.S. Zakordonets and G.N. Logvinov, Soviet Physics-Semic. **31**, 265 (1997)

[12] R. Venkatasubramanian and T. Colpitts, in *Thermoelectric Materials–New Directions and Approaches*, ed. T.M. Hicks, M.G. Kanatzidis, H.B. Lyons Jr, and G.D. Mahan (MRS, Vol. 478, 1997) pg 73

[13] E. T. Schwarz and R. O. Pohl, Appl. Phys. Lett. **51**, 2200 (1987)

[14] E.T. Schwartz and R.O. Pohl, *Rev. Mod. Phys.* **61**, 605 (89)

[15] S. M. Sze, *The Physics of Semiconductor Devices* 2nd ed.(Wiley, 1981)

[16] G. Springholz, G. Ihninger, G. Bauer, M.M. Olver, J.Z. Pastalan, S. Romaine and B.B. Goldberg, Appl. Phys. Lett. **63**, 2908 (1993)

[17] X.Y. Yu, G. Chen, A. Verma and J.S. Smith, Appl. Phys. Lett. **67**, 3554 (1995)

[18] G. Chen, DSC-Vol.59, Microelectromechanical Systems (MEMS), pg 13 (ASME, 1996)

[19] W.S. Capinski and H.J. Maris, Physica B **219 & 220**, 699 (1996)

[20] S.M. Lee, D.G. Cahill and R. Venkatasubramanian, Appl. Phys. Lett. **70**, 2957 (1997)

[21] P. Hyldgaard and G.D. Mahan, Phys. Rev. B **56**, 10 754 (1997)

[22] K. Kajiyama, M. Mizushima and S. Sakata, Appl. Phys. Lett. **23**, 458 (1973)

[23] L. Lerach and H. Albrecht, Surf. Sci. **78**, 531 (1978)

[24] L.D. Hicks, T.C. Harman, X. Sun, and M.S. Dresselhaus, Phys. Rev. B **53**, R10493 (1996)

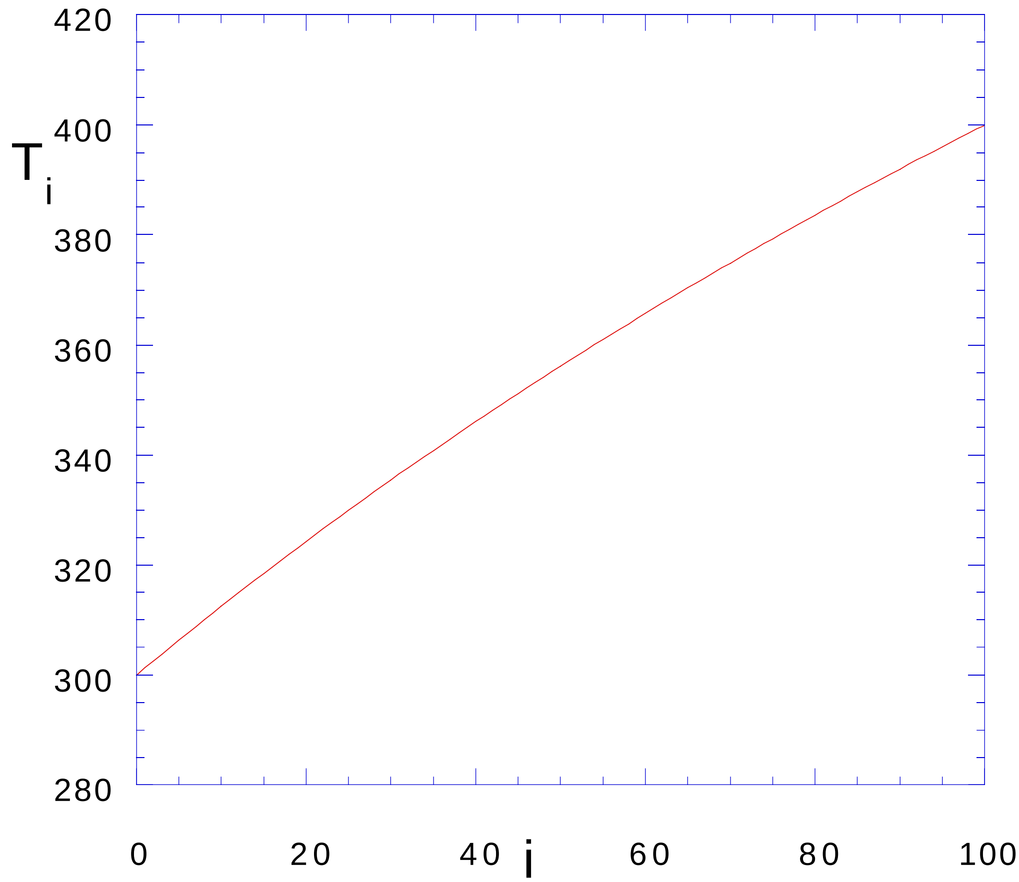
Figure Captions

1. The reduction in the Carnot efficiency $\hat{\eta}$ for a single barrier as a function of barrier height ϕ for values of T_R= 100, 200, 300, 400, and 500 K.

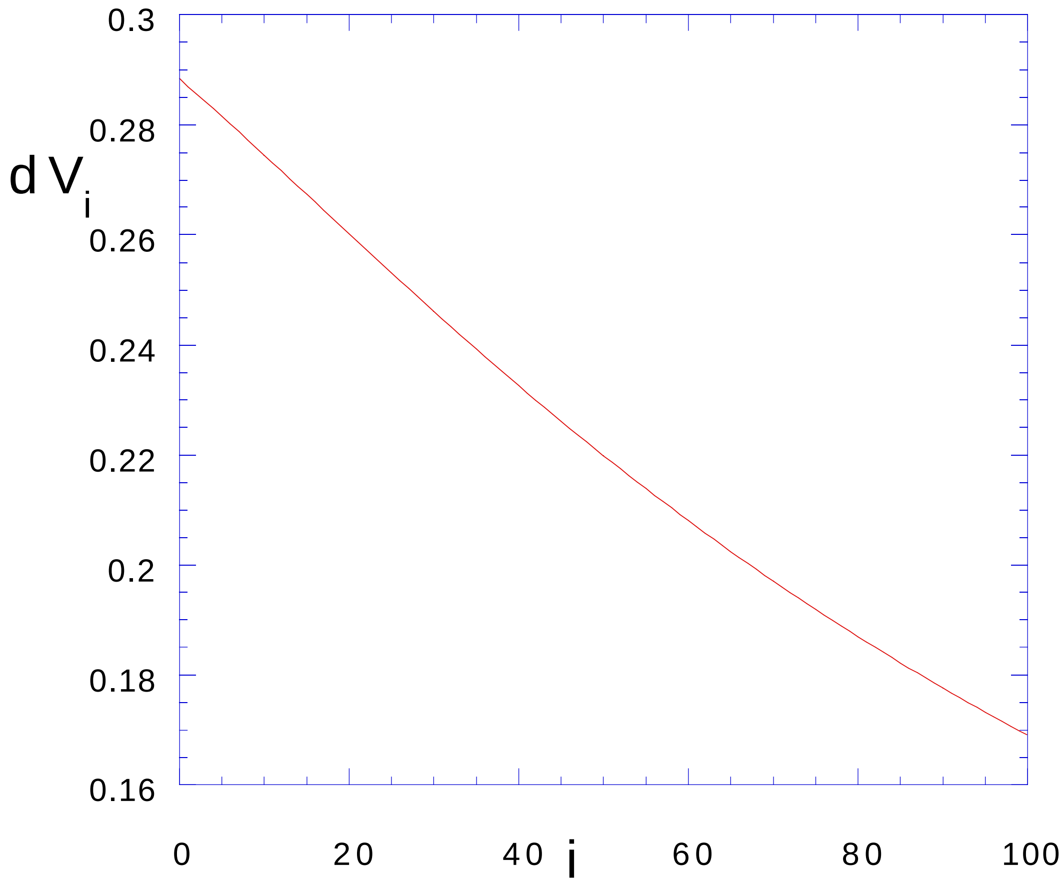
2. Efficiency η of a multilayer thermionic refrigerator with T_c = 260 K and T_h = 300 K. Results are plotted as a function of barrier height for different values of T_R.

3. Exact efficiency of a multilayer thermionic power generator as a function of barrier height ϕ for several values of T_R. We assumed T_c = 300 K and T_h = 400 K.

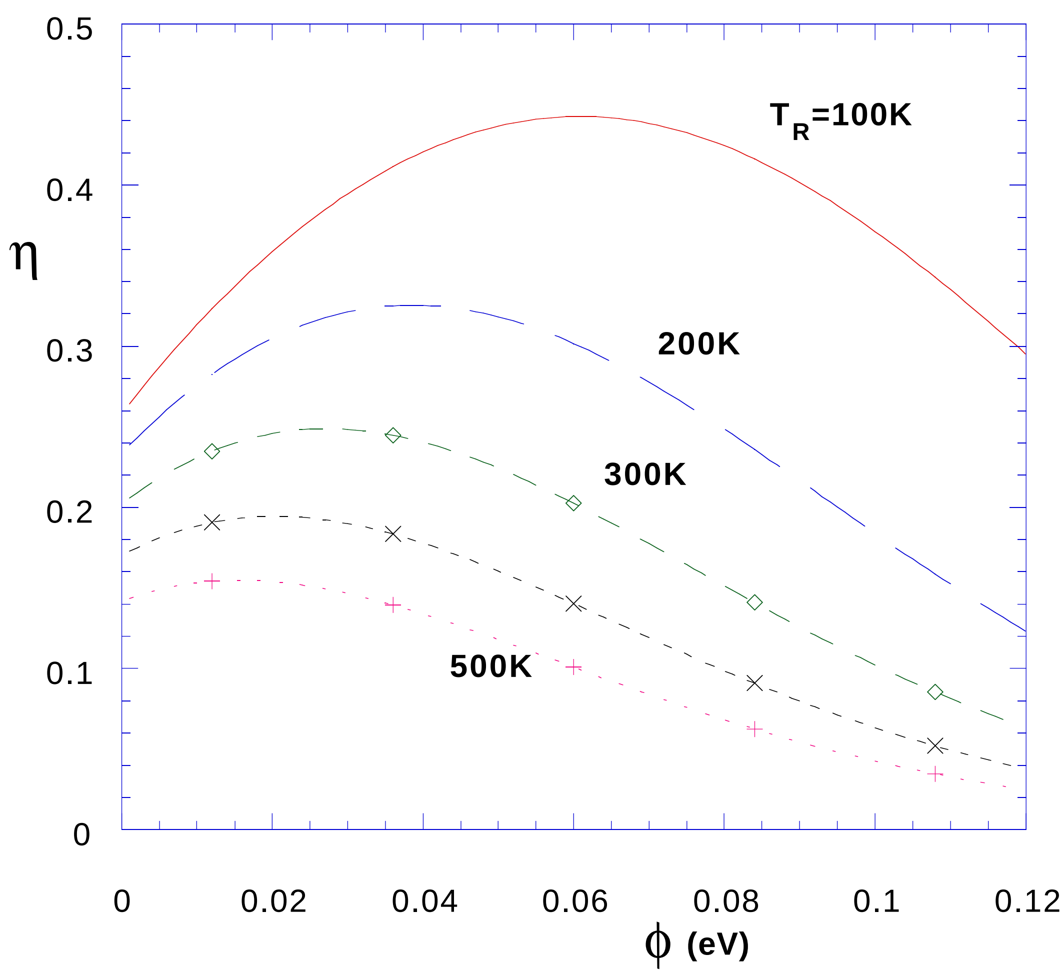
4. Change in the temperature T_i and voltage V_i as a function of position along the multilayer power generator. We assumed that T_c = 300 K, T_h = 400 K, ϕ = 0.054 V, and T_R = 200 K.



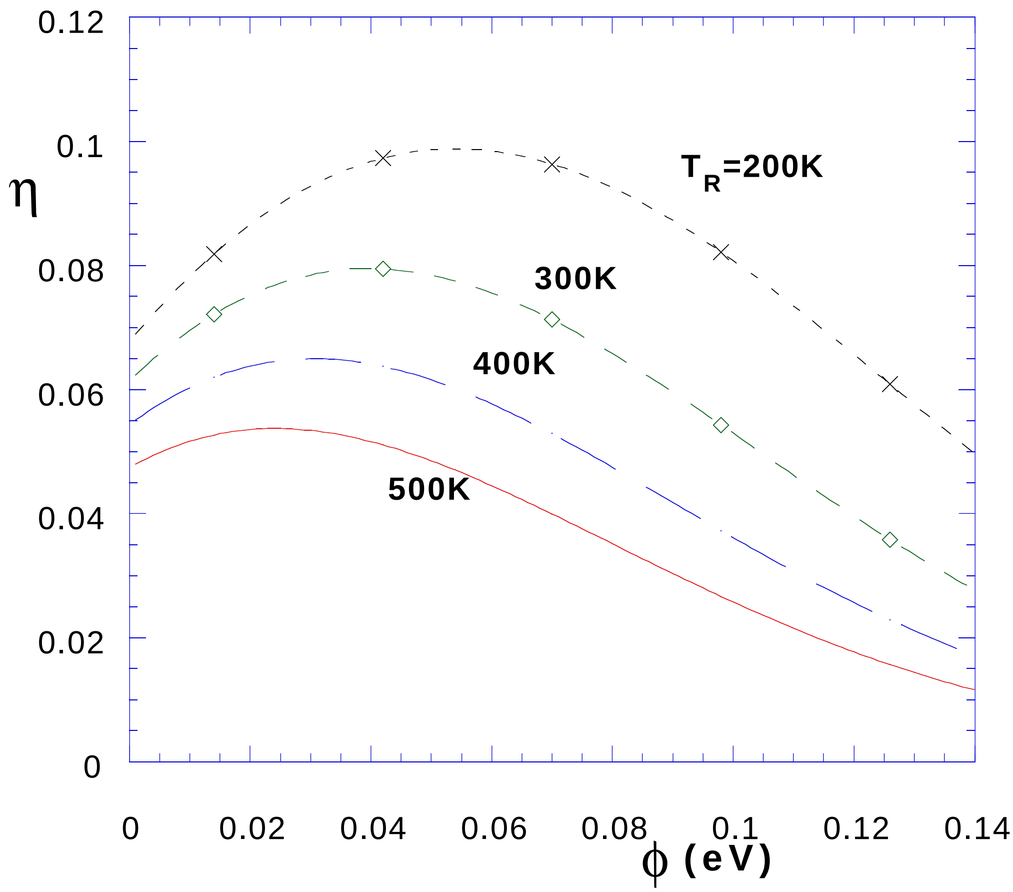
420
400
380
360
340
320
300
280
T_i
0
20
40
60
80
100
i



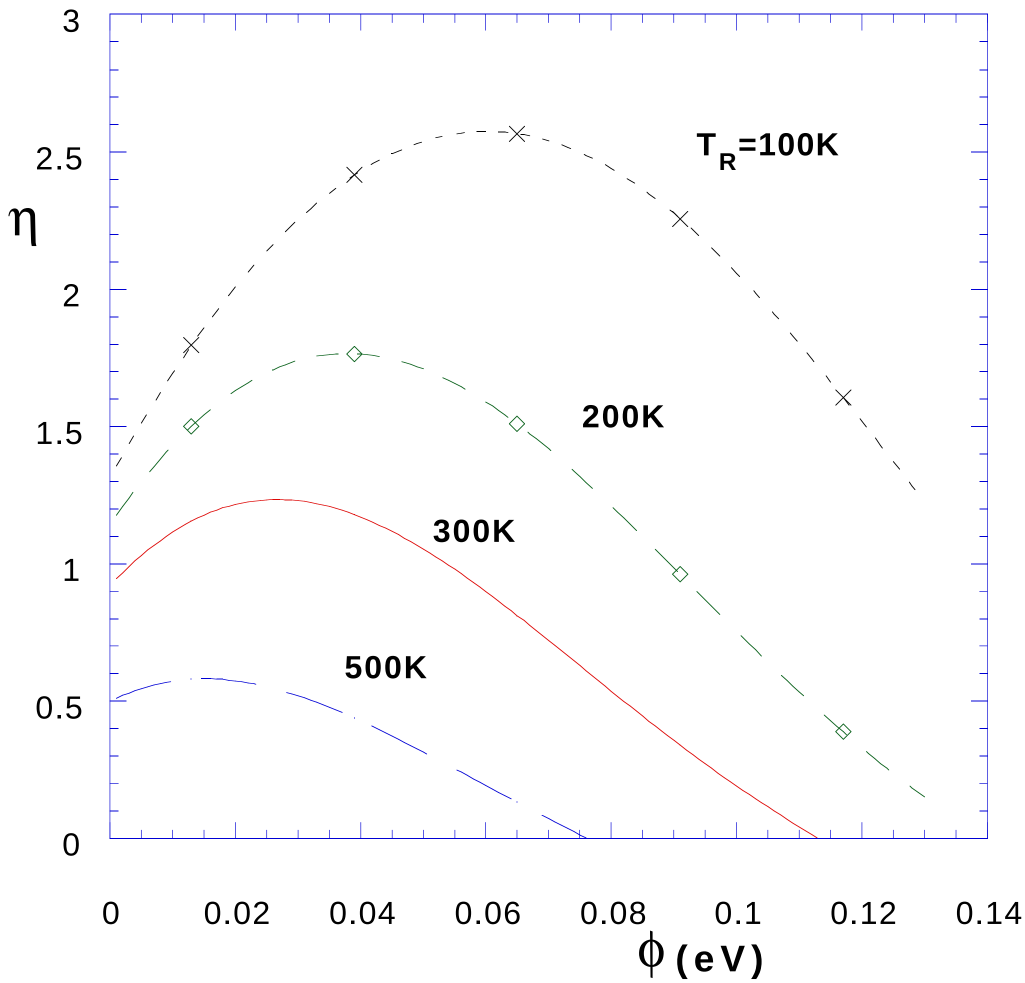
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
d V
i
0
20
40
60
80
100
i



0.5
0.4
0.3
0.2
0.1
0
η
T_R=100K
200K
300K
500K
0
0.02
0.04
0.06
0.08
0.1
0.12
φ (eV)



0.12
0.1
0.08
0.06
0.04
0.02
0
η
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
ϕ (eV)
T_R=200K
300K
400K
500K



3
2.5
2
1.5
1
0.5
0
η
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
φ (eV)
T_R=100K
200K
300K
500K